

Answer to Assignment 2

Problem 1

Note that for this problem, we can just use the formulas for demand with Cobb-Douglas utility:

$$x_1 = \frac{a}{a+b} \cdot \frac{m}{p_1} = \frac{4m}{5p_1} \quad \text{and} \quad x_2 = \frac{b}{a+b} \cdot \frac{m}{p_2} = \frac{m}{5p_2}$$

While the utility function we're given, $U(x_1, x_2) = 4 \ln x_1 + \ln x_2$, is not Cobb-Douglas, we can always take a monotonic transformation of it and obtain the same answers. Here, let $f(u) = e^u$ be the transformation so we're working instead with $U(x_1, x_2) = x_1^4 x_2$.

(a) With initial prices $p_1 = 10$, $p_2 = 1$, and income $m = 100$, we get demand

$$x_1(p_1, p_2, m) = \frac{4m}{5p_1} = 8 \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{m}{5p_2} = 20 .$$

With new price $p'_1 = 5$ we have that

$$x_1(p'_1, p_2, m) = \frac{4m}{5p'_1} = 16 \quad \text{and} \quad x_2(p'_1, p_2, m) = \frac{m}{5p_2} = 20 .$$

So the change in the number of movies (x_1) watched is

$$\Delta x_1 = x_1(p'_1, p_2, m) - x_1(p_1, p_2, m) = 16 - 8 = 8 .$$

(b) Movies are ordinary goods since the number demanded increases when its price decreased (demand is downward sloping).

(c) To find the substitution effect, we first need to find m' , which is the amount of income that would be required to purchase the $(x_1, x_2) = (8, 20)$ bundle (which was optimal at the original prices) at the new price p'_1 :

$$m' = p'_1 x_1 + p_2 x_2 = 5 \cdot 8 + 1 \cdot 20 = 60 .$$

So to purchase the original bundle at the new prices would require an income of $m' = \$60$ (rather than \$100). To find what amount of the total change in demand can be attributed to the substitution effect, we need to find the quantity demanded with income $m' = 60$ and price $p_1 = 5$:

$$x_1(p'_1, p_2, m') = \frac{4m'}{5p'_1} = 9\frac{3}{5} .$$

So we have that the substitution effect is

$$\Delta x_1^s = x_1(p'_1, p_2, m') - x_1(p_1, p_2, m) = 9\frac{3}{5} - 8 = 1\frac{3}{5} .$$

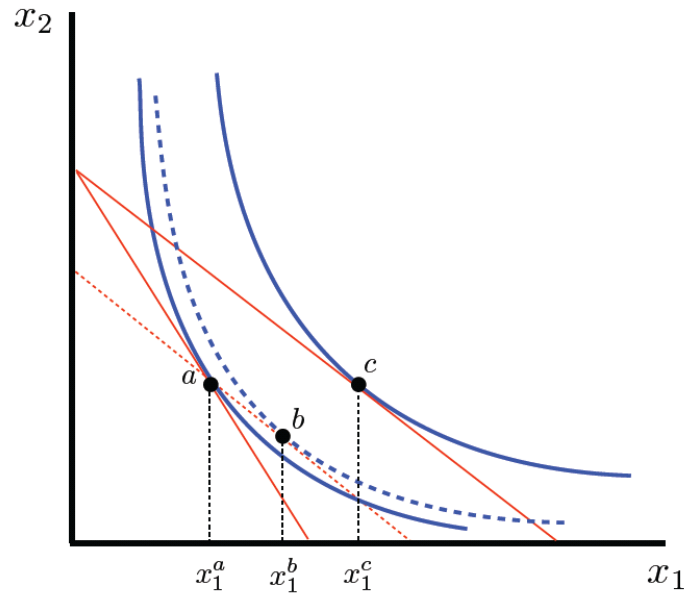
(d) The income effect is

$$\Delta x_1^n = x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m') = 16 - 9\frac{3}{5} = 6\frac{2}{5} .$$

(Of course, since $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ we can get this from $8 = 1\frac{3}{5} + \Delta x_1^n \implies \Delta x_1^n = 6\frac{2}{5}$ as well.)

(e) The income effect is positive (with prices p'_1 and p_2 , less of x_1 was demanded with the lower hypothetical m' than was demanded at income m). This means preferences over these goods are normal, which is true with Cobb-Douglas preferences.

(f) These effects are shown graphically below. Point a represents the initial bundle, where demand for x_1 is $x_1(p_1, p_2, m) = 8$. Point c represents demand after the price change where we had $x_1(p'_1, p_2, m) = 16$. Point b represents the optimal (and hypothetical) value with p'_1 and m' , with x_1 demand $x_1(p'_1, p_2, m') = 9\frac{3}{5}$. The difference between a and c is the total effect; between a and b is the substitution effect; between b and c is the income effect.



Problem 2

Recall that Trevor's preferences are represented by the utility function $U(x_1, x_2) = \min\{5x_1, x_2\}$ (He consumes five times as many strawberries (x_2) as he does milk (x_1) in making the perfect strawberry milkshake.) We found that his demand functions are given by:

$$x_1 = \frac{m}{p_1 + 5p_2} \quad \text{and} \quad x_2 = \frac{5m}{p_1 + 5p_2}.$$

(a) With $m = 200$, $p_1 = 15$, and $p_2 = 1$, his demands for milk (x_1) and strawberries (x_2) are:

$$x_1(p_1, p_2, m) = \frac{m}{p_1 + 5p_2} = \frac{200}{15 + 5 \cdot 1} = 10 \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{5m}{p_1 + 5p_2} = \frac{5 \cdot 200}{15 + 5 \cdot 1} = 50.$$

With the price of milk decreasing to $p'_1 = 5$, his new quantities demanded become:

$$x_1(p'_1, p_2, m) = \frac{m}{p'_1 + 5p_2} = \frac{200}{5 + 5 \cdot 1} = 20 \quad \text{and} \quad x_2(p'_1, p_2, m) = \frac{5m}{p'_1 + 5p_2} = \frac{5 \cdot 200}{5 + 5 \cdot 1} = 100.$$

The total change in the demand for milk is

$$\Delta x_1 = x_1(p'_1, p_2, m) - x_1(p_1, p_2, m) = 20 - 10 = 10.$$

(b) The substitution effect, Δx_1^s , for complementary goods is always zero. Let's verify this.

First, we need to find the m' income that would make the original bundle just affordable with the price change:

$$m' = p'_1 x_1 + p_2 x_2 = 5 \cdot 10 + 1 \cdot 50 = 100.$$

Next we find the quantity of x_1 demanded with $m' = 100$, $p'_1 = 5$ and $p_2 = 1$:

$$x_1(p'_1, p_2, m') = \frac{m'}{p'_1 + 5p_2} = \frac{100}{5 + 5 \cdot 1} = 10.$$

So we have that the substitution effect is

$$\Delta x_1^s = x_1(p'_1, p_2, m') - x_1(p_1, p_2, m) = 10 - 10 = 0.$$

(c) The income effect is

$$\Delta x_1^n = x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m') = 20 - 10 = 10.$$

(Again, since $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ we can get this from $10 = 0 + \Delta x_1^n \implies \Delta x_1^n = 10$ as well.)

Problem 3

(a) The MRS for these preferences is $MRS(x_1, x_2) = -\frac{5}{x_1}$. (Notice that this does not depend on x_2 , over which utility is linear.) The two secrets of happiness for an interior solution (with $x_1 > 0$ and $x_2 > 0$) are:

- (1) $p_1 x_1 + p_2 x_2 = m$
- (2) $MRS(x_1, x_2) = -\frac{p_1}{p_2} \implies -\frac{5}{x_1} = -\frac{p_1}{p_2}$

(b) Solving for the two above equations for x_1 and x_2 we get demand functions

$$x_1 = \frac{5p_2}{p_1} \quad \text{and} \quad x_2 = \frac{m - 5p_2}{p_2}$$

as long as $m \geq 5p_2$. (If $m < 5p_2$ we have a corner solution with $x_1 = \frac{m}{p_1}$ and $x_2 = 0$.)

(c) Given that $m = 10$ and $p_2 = 1$, at price $p_1 = 5$, our Miriam's demand for coffee x_1 is $x_1(p_1, p_2, m) = \frac{5 \cdot 1}{5} = 1$. (Also, $x_2(p_1, p_2, m) = \frac{10 - 5 \cdot 1}{1} = 5$.)

With a price of $p'_1 = 1$, her demand is $x_1(p'_1, p_2, m) = \frac{5 \cdot 1}{1} = 5$.

The total effect on demand for x_1 resulting from the price change is $\Delta x_1 = x_1(p'_1, p_2, m) - x_1(p_1, p_2, m) = 5 - 1 = 4$.

To find the substitution effect, we first find $m' = p'_1 x_1 + p_2 x_2$, the hypothetical income that would be necessary for her to buy her initial bundle with the price change, which is $m' = 1 \cdot 1 + 1 \cdot 5 = 6$.

Next we need to find the bundle Miriam would actually choose if her income were $m' = 6$. (We know she can afford the initial bundle, but that's not necessarily what she buys given the new relative prices.) At $p'_1 = 1$, $p_2 = 1$ and $m' = 6$, we have that $x_1(p'_1, p_2, m') = \frac{5 \cdot 1}{1} = 5$.

The substitution effect is then

$$\Delta x_1^s = x_1(p'_1, p_2, m') - x_1(p_1, p_2, m) = 5 - 1 = 4 .$$

(d) The income effect on the demand for x_1 is

$$\Delta x_1^n = x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m') = 5 - 5 = 0 .$$

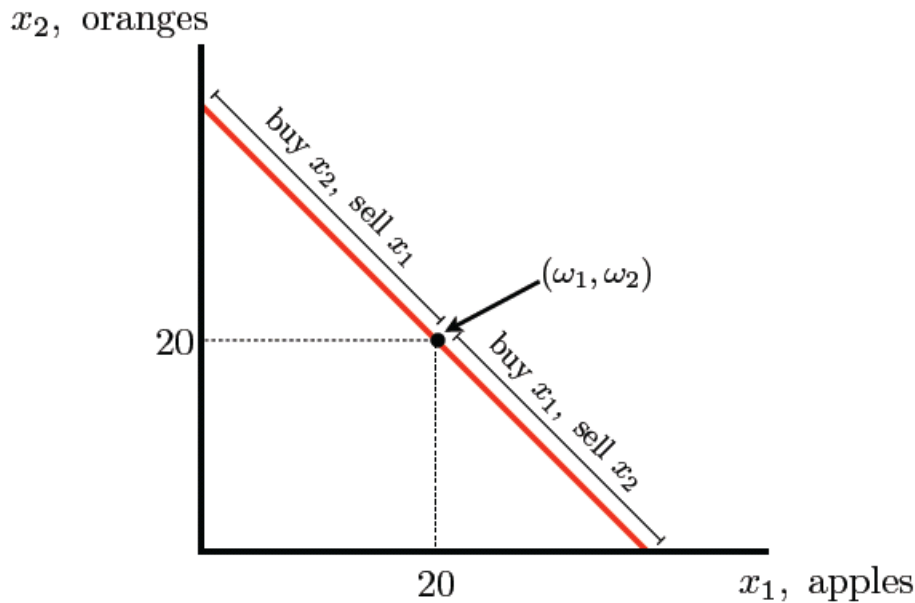
(Again, since $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ we can get this from $4 = 4 + \Delta x_1^n \implies \Delta x_1^n = 0$ as well.) With quasilinear preferences, the good in which utility is linear (here, x_2) “absorbs all income effects.”

Problem 4

(a) Let x_1 be the consumption of apples and x_2 be the consumption of oranges. Dave's budget constraint is given by

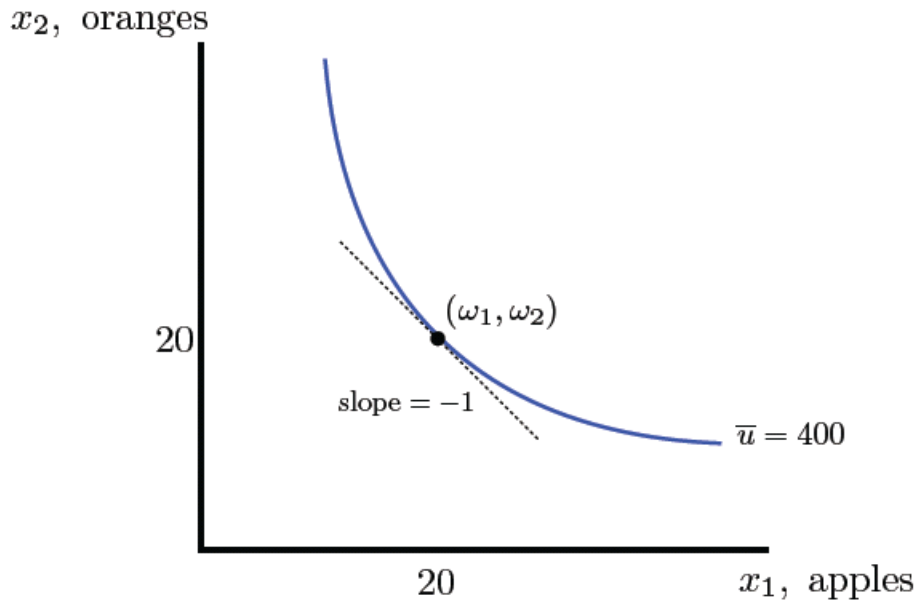
$$p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2 \implies 2x_1 + 2x_2 \leq 80 .$$

The bundles for which Dave would be selling apples and buying oranges is shown below:



(b) First, the MRS of $U(x_1, x_2) = x_1 x_2$ is $MRS(x_1, x_2) = -\frac{x_2}{x_1}$. At his endowment point $x_1 = \omega_1 = 20$ and $x_2 = \omega_2 = 20$, we have $MRS(20, 20) = -\frac{20}{20} = -1$.

The utility associated with this bundle is $U(20, 20) = 20 \cdot 20 = 400$. Dave is indifferent then among his endowment and all the other bundles for which $x_1 x_2 = 400$ so the indifference curve passing through his endowment is, analytically, $x_2 = 400/x_1$. This is shown graphically below:



(c) Demand with these preferences is

$$x_1(p_1, p_2, m) = \frac{m}{2p_1} \quad \text{and} \quad x_2(p_1, p_2, m) = \frac{m}{2p_2}.$$

Holding $p_2 = 2$ fixed, for the three different p_1 prices (and resulting m changes), we have the following demand:

- (1) $p_1 = 1$, $m = 1 \cdot 20 + 2 \cdot 20 = 60$: $x_1(1, 2, 60) = \frac{60}{2 \cdot 1} = 30$ and $x_2(1, 2, 60) = \frac{60}{2 \cdot 2} = 15$
 - Net demand for x_1 is given by $x_1(1, 2, 60) - \omega_1 = 30 - 20 = 10$
 - Net demand for x_2 is zero since $x_2(1, 2, 60) = 15 < 20 = \omega_2$. (He will be a net supplier of x_2 .)
- (2) $p_1 = 2$, $m = 2 \cdot 20 + 2 \cdot 20 = 80$: $x_1(2, 2, 80) = \frac{80}{2 \cdot 2} = 20$ and $x_2(2, 2, 80) = \frac{80}{2 \cdot 2} = 20$
 - Net demand for x_1 is given by $x_1(2, 2, 80) - \omega_1 = 20 - 20 = 0$. (He is neither a net supplier for nor a net demander of x_1 .)
 - Net demand for x_2 is given by $x_2(2, 2, 80) - \omega_2 = 20 - 20 = 0$. (He is neither a net supplier for nor a net demander of x_2 .)
- (3) $p_1 = 3$, $m = 3 \cdot 20 + 2 \cdot 20 = 100$: $x_1(3, 2, 100) = \frac{100}{2 \cdot 3} = \frac{50}{3}$ and $x_2(3, 2, 100) = \frac{100}{2 \cdot 2} = 25$
 - Net demand for x_1 is zero since $x_1(3, 2, 100) = \frac{50}{3} > 20 = \omega_1$. (He will be a net supplier of x_1 .)
 - Net demand for x_2 is given by $x_2(3, 2, 100) - \omega_2 = 25 - 20 = 5$

(d) The magnitude of the slope of the indifference curve at the endowment point is $|MRS(20, 20)| = |-\frac{20}{20}| = 1$.

- (1) For $p_1 = 1$, $|MRS(20, 20)| = 1 > \frac{1}{2} = |-\frac{p_1}{p_2}|$. He is better off buying more apples (x_1) and selling some of his oranges (x_2).
- (2) At $p_1 = 2$, $|MRS(20, 20)| = 1 = 1 = |-\frac{p_1}{p_2}|$. He can not get any greater utility from buying or selling apples and oranges; he is best off with his endowment.
- (3) At $p_1 = 3$, $|MRS(20, 20)| = 1 < \frac{3}{2} = |-\frac{p_1}{p_2}|$. He is better off buying more oranges (x_2) and selling some of his apples (x_1).

(e) Dave's optimal bundles for the three price combinations found above analytically are shown graphically below with bundles (1), (2), and (3) labeled. The price-offer curve must pass through these points, and is shown in the second graph. You will see that at all but one point (the endowment, point (2) = (ω_1, ω_2)), the price offer curve lies strictly above the indifference curve passing through the endowment (where $\bar{u} = 400$). This is significant: It means that one is always at least as well off trading as he is just consuming his endowment!

